

Initial state radiation corrections to double charmonium production in one-photon electron-positron annihilation

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(Dated: February 1, 2008)

In this paper initial state radiation corrections to double production of charmonium mesons on one-photon electron-positron annihilation at center of mass energy $\sqrt{s} = 10.6$ GeV are studied. It is shown, that these corrections have noticeable effect and must be taken into consideration.

PACS numbers: 14.40.Gx, 13.66.Bc

I. INTRODUCTION

Since the discovery of the so called charmonium mesons (that is mesons consisting of c - and \bar{c} -quarks, for example J/ψ , $\psi(2S)$ or η_c) in 1974 [1, 2] they play a significant role in our understanding of quantum chromodynamics (QCD). Production of charmonium requires creation of heavy $c\bar{c}$ pair with the energy greater than $2m_c$ where QCD coupling constant is small enough to use perturbation theory. However, the subsequent hadronization probes much smaller mass scales of order $m_c v^2$, where v is typical velocity of c quark in the charmonium rest frame. For J/ψ , $m_c v^2$ is numerically of order Λ_{QCD} , so the production processes are sensitive to nonperturbative physics as well. Charmonium mesons are also very interesting from the experimental point of view, since the narrow resonances can easily be separated from the background.

In 1986 Caswell and Lepage have proposed the Non-Relativistic Quantum Chromodynamics model (NRQCD) [3, 4]. This model exploits the fact that the velocity v of the heavy quark in the charmonium rest frame is small in comparison with the speed of light and all physical quantities can be expressed as the series in this small velocity and electromagnetic and strong coupling constants α and α_s . Matrix elements of the four-fermion operators, used in this theory, can be determined phenomenologically. Both inclusive and exclusive production of charmonium mesons and their decays were studied using this theory [5, 6, 7, 8, 9, 10, 11].

New difficulties have arisen as a result of recent measurements of inclusive charmonium production in e^+e^- collisions by Belle and BABAR collaborations [12, 13]. Their results were about an order of magnitude higher, than the predictions based on NRQCD or other models [5, 14, 15, 16, 17].

Another difference has arisen in studying the exclusive production of pair of charmonium mesons. For example, the NRQCD results for $J/\psi\eta_c$ production is [8]

$$\sigma_0 = \sigma(e^+e^- \rightarrow \gamma^* \rightarrow J/\psi\eta_c) = 2.31 \text{ fb}$$

and Belle results [18] are an order of magnitude higher. One of the possible explanations of this difference was proposed in [19]. The authors assumed that some of the Belle's $J/\psi\eta_c$ signals could actually be the double production of J/ψ meson with subsequent decay $J/\psi \rightarrow \eta_c\gamma$ and presented the value

$$\sigma(e^+e^- \rightarrow 2\gamma^* \rightarrow 2J/\psi) = 8.7 \text{ fb}.$$

Later in [8, 20] it was shown that due to relativistic and higher order QCD corrections the cross section should be about 4 times smaller and the question of the difference between theory and experiment remains open.

In this paper I propose that this difference can be explained by the initial state radiation corrections. Indeed, the naive estimate of the effect of these corrections gives us the result

$$\sigma = \sigma(e^+e^- \rightarrow J/\psi\eta_c\gamma) \sim \alpha \log \frac{s}{m_e^2} \sigma_0$$

and the suppression caused by additional factor α will be compensated by large logarithm $\log s/m_e^2$. Another reason is that cross section of the reaction $e^+e^- \rightarrow J/\psi\eta_c$ as a function of center of mass energy \sqrt{s} has a narrow peak near $\sqrt{s} \approx 6.5$ GeV, where $\sigma(e^+e^- \rightarrow J/\psi\eta_c) \approx 30 \text{ fb}$ and by emitting a hard photon we can return to this region.

The rest of this paper is organized as follows. In the next section I give the diagrams and matrix element for the process under consideration. In section III the method of calculation of the total cross section is described. Finally, distributions over vector charmonium energy and scattering angle are presented.

II. MATRIX ELEMENT

The diagrams for the process $e^-(k_1)e^+(k_2) \rightarrow J/\psi(p_1, E)\eta_c(p_2)\gamma(k, \epsilon)$ are shown on figure 1. The vertex of the transition of virtual photon with momentum \hat{p} and its square $\hat{s} = \hat{p}^2$ into pair of charmonium states, denoted here by filled circle, was calculated in [8]. Some of the diagrams of this transition are shown on figure 2, the others can be obtained from them by permutations. One can also add electromagnetic diagrams by replacing the gluon line in figure 2(a) by photon, but they are

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suppressed by the factor α/α_s and will not be considered here. On the other hand, as it was shown in [8] the purely electromagnetic resonance diagrams (for example, the diagram 2(b)) must be included. The reason for this is that the additional factor α is compensated by $r^2 = 4M^2/\hat{s}$, where $M = 2m_c$ is the mass of the charmonium meson (the difference between J/ψ and η_c masses is neglected) and m_c is c -quark mass.

The analytical expression for the $\gamma J/\psi \eta_c$ vertex was found to be

$$\begin{aligned} J_\mu &= \langle J/\psi(p_1, \varepsilon) \eta_c(p_2) | \bar{c}\gamma_\mu c | 0 \rangle = \\ &= iA\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu p_1^\alpha p_2^\beta, \end{aligned}$$

where ε is the polarization vector of the vector charmonium J/ψ , the coefficient A is

$$A = \frac{128\pi\alpha_s}{\hat{s}^2} \sqrt{O_\psi O_\eta} \left(\frac{N_c^2 - 1}{2N_c^2} + \frac{e_c^2\alpha}{N_c\alpha_s} - \frac{1}{r^2} \frac{e_c^2\alpha}{\alpha_s} \right),$$

$N_c = 3$ is the number of colors, $e_c = 2/3$ is the charge of the c quark (in the units of the elementary charge e), and the constants

$$O_\psi = \langle O_1 \rangle_{J/\psi}, \quad O_\eta = \langle O_1 \rangle_{\eta_c}$$

are the matrix elements of the NRQCD probability factors. These constants can be determined from the experimental data on J/ψ and η_c decay widths [8]:

$$\begin{aligned} O_\psi &= \frac{27}{32\pi\alpha^2} \Gamma(J/\psi \rightarrow e^+e^-) M^2 \left(1 - \frac{8}{3} \frac{\alpha_s}{\pi} \right)^{-2}, \\ O_\eta &= \frac{81}{128\pi\alpha^2} \Gamma(\eta_c \rightarrow \gamma\gamma) \left(1 - \frac{2 - \pi^2}{6} \frac{\alpha_s}{\pi} \right)^{-2}. \end{aligned}$$

Using the table data on particle decay widths [21] we get the numerical values of probability factors, that will be used in this paper:

$$\begin{aligned} O_\psi &= 0.335 \text{ GeV}^3, \\ O_\eta &= 0.297 \text{ GeV}^3. \end{aligned}$$

The matrix element corresponding to the diagrams shown on figure 1 is

$$\mathcal{M} = \frac{e_ce^3}{\hat{s}} \epsilon^\mu J^\nu \bar{v}(k_2) \left\{ \frac{1}{q_a^2} \gamma_\nu \hat{q}_a \gamma_\mu + \frac{1}{q_b^2} \gamma_\mu \hat{q}_b \gamma_\nu \right\} u(k_1), \quad (1)$$

where k_1 and k_2 are electron and positron momenta (the mass of the electron m_e is neglected wherever it is possible), k is the momentum of the final state photon, ϵ_μ is its polarization vector, $q_a = k_1 - k$ and $q_b = k - k_2$ are the momentum transfer in this two diagrams. The cross section found from (1) and equals to

$$\sigma = \frac{1}{2048\pi^7 (k_1 k_2)} \int d\Phi_3(k_1 + k_2; p_1, p_2, k) \sum |\mathcal{M}|^2,$$

where summation is performed over polarizations of initial and final particles and for the Lorentz-invariant phase space the notation

$$d\Phi_n(P; p_1, \dots, p_n) = \delta \left(P - \sum_{i=1}^n p_i \right) \prod_{i=1}^n \frac{d^3 p_i}{2E_i} \quad (2)$$

is used.

III. TOTAL CROSS SECTION

The Lorentz-invariant phase space (2) satisfies the recursion relation

$$\begin{aligned} d\Phi_3(k_1 + k_2; p_1, p_2, k) &= d\hat{s} d\Phi_2(k_1 + k_2; \hat{p}, k) \\ &\times d\Phi_2(\hat{s}; p_1, p_2), \end{aligned}$$

where $\hat{p} = p_1 + p_2$ is the momentum of the virtual photon, $\hat{s} = \hat{p}^2 = s - 2\sqrt{s}E_k$, and E_k is the final state photon energy in the laboratory frame. Leaving only the integration over E_k we get the differential cross section

$$\begin{aligned} \frac{d\sigma}{d\hat{s}} &= \frac{32768\pi^2\alpha^3}{19683} \frac{O_\psi O_\eta}{\hat{s}^2} \left(1 - \frac{4M^2}{\hat{s}} \right)^{3/2} \times \\ &\times \frac{(2\alpha\hat{s} - 4M^2(\alpha + 2\alpha_s))^2}{\hat{s}^4} \times \\ &\times \left(2 \frac{s^2 + \hat{s}^2}{s - \hat{s}} \frac{\operatorname{arctanh} \beta_e}{\beta_e} - s + \hat{s} \right). \end{aligned}$$

From this equation one can easily notice two important items:

- the cross section is enhanced by the factor $\operatorname{arctanh} \beta_e \sim \log s/m_e^2$ as it was stated in the introduction,
- total cross section actually diverges in the limit $\hat{s} \approx s$. This fact is the familiar infrared divergence caused by the emission of the soft photon, where one can not use the perturbation theory. To avoid this divergence one must put the cutoff δ_ϵ on the energy of the photon:

$$E_k > \delta_\epsilon.$$

The plot for differential cross section $d\sigma/dE_k$ is presented on figure 3. As stated above, in the limit of small photon energy it grows to infinity and one should put the cutoff. The bump in the region $E_k \approx 3 \text{ GeV}$ corresponds to $\sqrt{\hat{s}} \approx 6.5 \text{ GeV}$, where the cross section of the process $\gamma^* \rightarrow J/\psi \eta_c$ is maximal.

Values of total cross section for some cutoff photon energies are:

$$\begin{aligned} \sigma &\approx 2.84 \text{ fb} \quad \text{for } \delta_\epsilon = 1 \text{ MeV}, \\ \sigma &\approx 2.234 \text{ fb} \quad \text{for } \delta_\epsilon = 10 \text{ MeV}, \\ \sigma &\approx 1.72 \text{ fb} \quad \text{for } \delta_\epsilon = 100 \text{ MeV}. \end{aligned}$$

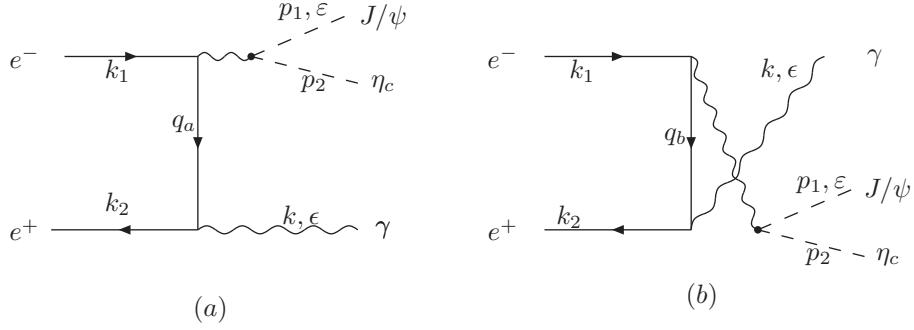
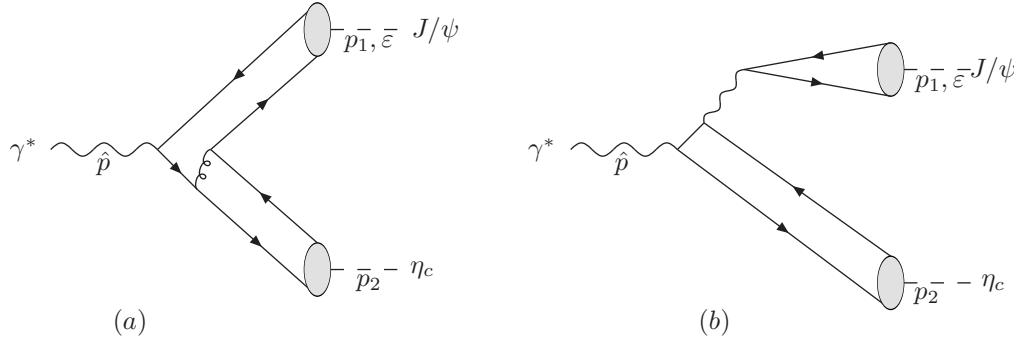
FIG. 1: Diagrams for $e^+e^- \rightarrow J/\psi\eta_c\gamma$.

FIG. 2: Some diagrams for the transition of a virtual photon into charmonium pair. Other diagrams can be obtained from these by permutations.

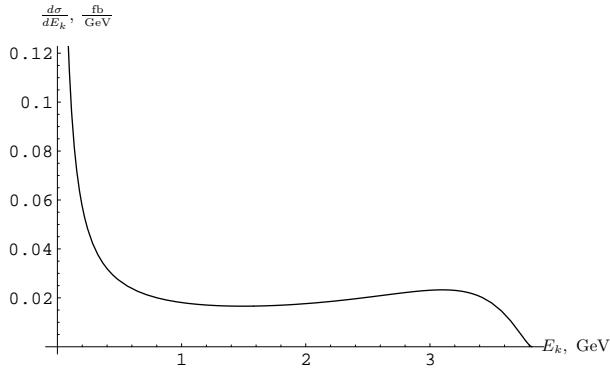


FIG. 3: Distribution over final state photon energy

It is easily seen, that by the order of magnitude these cross sections equals to the cross section of the base process $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi\eta_c$, so the initial state radiation corrections should be taken into account.

IV. DIFFERENTIAL CROSS SECTION

For the calculation of the differential cross section over the vector charmonium energy it is convenient to introduce dimensionless variables according to formula's

$$z_{k,1,2} = \frac{2E_{k,1,2}}{\sqrt{s}},$$

$$\begin{aligned} \mu &= \frac{2M}{\sqrt{s}}, \\ x_{k,1,2} &= \cos \theta_{k,1,2}, \\ y_{k,1,2} &= \sqrt{1 - x_{k,1,2}^2}, \end{aligned}$$

where E_1 , E_2 and E_k are the energies of vector and pseudoscalar charmonia and photon in the center of mass frame respectively; θ_1 , θ_2 and θ_k are the angles between the momentum of the initial electron and the momenta of these final particles. These variables take the values in the regions

$$\begin{aligned} x_1, x_k &\in [-1; 1], \\ z_1 &\in [\mu; 1], \\ z_k &\in [z_k^m; z_k^M], \end{aligned}$$

where minimum and maximum values of z_k are

$$z_k^{m,M} = \frac{2(1 - z_1)}{2 - z_1[1 - \beta(x_1x_k \pm y_1y_k)]}.$$

In the terms of these variables the squared matrix element and Lorentz-invariant phase space take the form

$$\begin{aligned}
|\mathcal{M}|^2 &= \frac{13417728\alpha^3}{6561\mu^4} \frac{O_\psi O_\eta}{s^4} \frac{(3\alpha_s\mu - \alpha(3 - 3z_k - \mu^2))^2}{z_k^2(1-z_k)^5(1-\beta_e^2x_k^2)} \left\{ 4(2 - \mu^2 - 2z_1 + (1 + \beta^2 x_1^2)z_1^2) - \right. \\
&\quad 4(4 - \mu^2 - 3(3 + \beta x_1 x_k) z_1 + (1 + \beta^2 x_1^2) z_1^2) z_k + \\
&\quad (2(5 - 2z_1(1 - \beta x_1 x_k) + x_k^2) - \mu^2(1 + x_k^2)) z_k^2 - \\
&\quad \left. 2(1 + x_k^2) z_k^3 \right\}, \\
d\Phi_3(k_1 + k_2; p_1, p_2, k) &= \frac{\pi s}{16} \frac{\beta z_1 z_k}{1 - z_1} \sqrt{\frac{z_k^M z_k^m}{(z_k^M - z_k)(z_k - z_k^m)}} dz_1 dx_1 dz_k dx_k,
\end{aligned}$$

where $\beta = \sqrt{1 - M^2/E_1^2} = \sqrt{1 - \mu^2/z_1^2}$ is the velocity of the vector charmonium meson and $\beta_e = \sqrt{1 - 4m_e^2/s}$ is the velocity of the initial electron in the laboratory frame (this is the only place where I have to leave the non-vanishing electron mass).

For integration of the differential cross section over z_k it is possible to use the residue technique. The only $d^4\sigma/dx_1 dz_1 dx_z dz_k$ poles are $z_k = 0, 1$ in , so we can write

$$\frac{d^3\sigma}{dx_1 dz_1 dx_k} = i\pi \sum_{z_k=0,1} \text{res} \frac{d^4\sigma}{dx_1 dz_1 dx_k dz_k}.$$

The residue at $z_k = 1$ is too lengthy to be presented here, but the residue at $z_k = 0$, that plays most important role for $z_1 \approx 1$ (that is in the limit of maximal vector charmonium energy) and $x_1 \neq \pm 1$ it is rather compact. In this limit we have

$$\begin{aligned}
\frac{d^2\sigma}{dz_1 dx_1} &= S(z_1) [1 + B(z_1)x_1^2], \\
S(z_1) &= \frac{262144\pi^2\alpha^3\beta}{6561\mu^4} \frac{O_\psi O_\eta}{s^4} \frac{\arctanh\beta_e}{\beta_e} z_1 \times \\
&\quad \times \frac{(2 - \mu^2 - 2z_1 + z_1^2)(3\alpha_s\mu^2 - \alpha(3 - \mu^2))^2}{1 - z_1},
\end{aligned}$$

$$B(z_1) = \frac{\beta^2 z_1^2}{2 - \mu^2 - 2z_1 + z_1^2}.$$

The integration over all other variables was performed numerically.

The distribution over vector charmonium energy is shown on figure 4. In the limit of maximal vector charmonium energy ($z_1 \approx 1$) this curve grows to infinity corresponding to analogous grows int the distribution over photon energy shown on figure 3.

The distributions over scattering angle for different values of δ_ϵ are shown on figure 5. This form of distribution is usual for color-singlet charmonium production.

Acknowledgments

Author would like to thank A.K. Likhoded for useful discussions. This work was partly supported by Russian Scientific School, grant #1202.2003.2.

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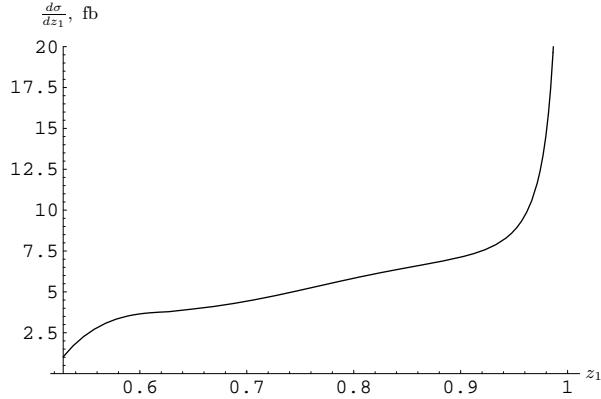


FIG. 4: Distribution over vector charmonium energy.

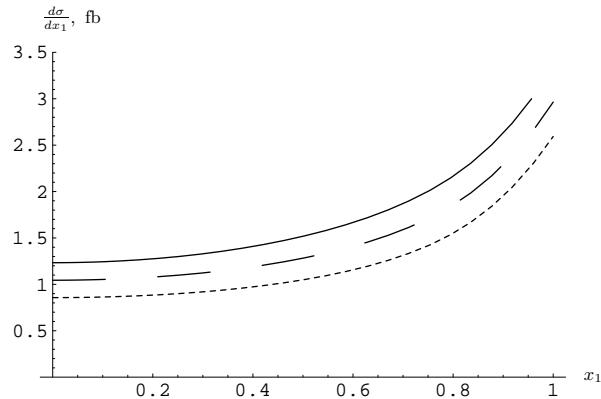


FIG. 5: Distribution over scattering angle for $\delta_\epsilon = 1 \text{ MeV}$ (solid line), $\delta_\epsilon = 10 \text{ MeV}$ (dashed line), and $\delta_\epsilon = 100 \text{ MeV}$ (dotted line)

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